

## 1. INTRODUCTION

Let  $X$ ,  $Y$  and  $Z$  be topological spaces. For a mapping  $f : X \times Y \rightarrow Z$  and a point  $(x, y) \in X \times Y$  we write  $f^x(y) = f_y(x) = f(x, y)$ . A mapping  $f : X \times Y \rightarrow Z$  is said to be *separately continuous*, if mappings  $f^x : Y \rightarrow Z$  and  $f_y : X \rightarrow Z$  are continuous for all  $x \in X$  and  $y \in Y$ . If  $f : X \times Y \rightarrow Z$  is a pointwise limit of a sequence of separately continuous mappings  $f_n : X \times Y \rightarrow Z$ , then  $f$  is a *Baire-one mapping* or  $f$  belongs to the *first Baire class*.

In 1898 H. Lebesgue [1] proved that if  $X = Y = \mathbb{R}$  then every separately continuous function  $f : X \times Y \rightarrow Z$  belongs to the first Baire class. A collection of topological spaces  $(X, Y, Z)$  with this property we call a *Lebesgue triple*.

The result of Lebesgue was generalized by many mathematicians (see [2, 3, 4, 5, 6] and the references given there). In particular, A. Kalancha and V. Maslyuchenko [4] showed that  $(\mathbb{R}, \mathbb{R}, Z)$  is a Lebesgue triple if  $Z$  is a topological vector space. T. Banach [5] proved that  $(\mathbb{R}, \mathbb{R}, Z)$  is a Lebesgue triple in the case when  $Z$  is an equiconnected space. It follows from [6, Theorem 3] that for every metrizable arcwise connected and locally arcwise connected space  $Z$  a collection  $(\mathbb{R}, \mathbb{R}, Z)$  is a Lebesgue triple.

In the connection with the above-mentioned results V. Maslyuchenko put the following question.

**Question 1.1.** *Does there exist a topological space  $Z$  such that  $(\mathbb{R}, \mathbb{R}, Z)$  is not a Lebesgue triple?*

Here we give the positive answer to this question. Moreover, we prove that  $(X, Y, Z)$  is not a Lebesgue triple for topological spaces  $X$  and  $Y$  of a wide class of spaces, which includes, in particular, all spaces  $\mathbb{R}^n$ , and for a space  $Z = X \times Y$  endowed with the cross-topology (see definitions in Section 2). In Sections 2 and 3 we give some auxiliary properties of this topology. Section 4 contains a proof of the main result. In the last section we show that connectedness-like conditions on spaces  $X$  and  $Y$  in the main result are essential. We prove that  $(X, Y, Z)$  is a Lebesgue triple when  $X$  is a strongly zero-dimensional metrizable space,  $Y$  and  $Z$  are arbitrary topological spaces.

## 2. COMPACT SETS IN THE CROSS-TOPOLOGY

Let  $X$  and  $Y$  be topological spaces. We denote by  $\gamma$  the collection of all subsets  $A$  of  $X \times Y$  such that for every point  $(x, y)$  of  $A$  there exist such neighborhoods  $U$  and  $V$  of  $x$  and  $y$  in  $X$  and  $Y$ , respectively, that  $(\{x\} \times V) \cup (U \times \{y\}) \subseteq A$ . The system  $\gamma$  forms a topology on  $X \times Y$ , which is called *the cross-topology*. A product  $X \times Y$  with the cross-topology we denote by  $(X \times Y, \gamma)$ .

For a point  $p = (x, y) \in X \times Y$  by  $\text{cross}(p)$  we denote the set  $(\{x\} \times Y) \cup (X \times \{y\})$ . For an arbitrary  $A \subseteq X \times Y$  let  $\text{cross}(A) = \bigcup_{p \in A} \text{cross}(p)$ .

**Proposition 2.1.** *Let  $X$  and  $Y$  be  $T_1$ -spaces and  $(p_n)_{n=1}^\infty$  a sequence of points  $p_n = (x_n, y_n) \in X \times Y$  such that  $x_n \neq x_m$  and  $y_n \neq y_m$  if  $n \neq m$ . Then  $P = \{p_n : n \in \mathbb{N}\}$  is a  $\gamma$ -discrete set.*

*Proof.* Since every one-point subset of  $X$  or of  $Y$  is closed,  $P$  is  $\gamma$ -closed. Similarly, every subset  $Q \subseteq P$  is also  $\gamma$ -closed. Hence,  $P$  is closed discrete subspace of  $(X \times Y, \gamma)$ .  $\square$

**Proposition 2.2.** *Let  $X$  and  $Y$  be  $T_1$ -spaces and let  $K \subseteq X \times Y$  be a  $\gamma$ -compact set. Then there exists a countable set  $A \subseteq X \times Y$  such that  $K \subseteq \text{cross}(A)$ .*

*Proof.* Assume that  $K \not\subseteq \text{cross}(A)$  for any finite set  $A \subseteq X \times Y$ . We choose an arbitrary point  $p_1 \in K$  and by the induction on  $n \in \mathbb{N}$  we construct a sequence of points  $p_n \in K$  such that  $p_{n+1} \in K \setminus \text{cross}(P_n)$ , where  $P_n = \{p_k : 1 \leq k \leq n\}$  for every  $n \in \mathbb{N}$ . According to Proposition 2.1 the set  $P = \{p_n : n \in \mathbb{N}\}$  is infinite  $\gamma$ -discrete in  $K$  which contradicts the fact that  $K$  is  $\gamma$ -compact.  $\square$

**Proposition 2.3.** *Let  $X$  and  $Y$  be  $T_1$ -spaces and let  $A$  and  $B$  be discrete sets in  $X$  and  $Y$ , respectively. Then the topology of product  $\tau$  and the cross-topology  $\gamma$  coincide on the set  $C = \text{cross}(A \times B)$ .*

*Proof.* Fix  $p = (x, y) \in C$ . Using the discreteness of  $A$  and  $B$ , we choose such neighborhoods  $U$  and  $V$  of  $x$  and  $y$  in  $X$  and  $Y$ , respectively, that  $|U \cap A| \leq 1$  and  $|V \cap B| \leq 1$ . Then  $C \cap (U \times V) = C \cap \text{cross}(c)$  for some point  $c \in C$ . Then  $\tau = \gamma$  on the set  $C \cap (U \times V)$ .  $\square$

Propositions 2.2 and 2.3 immediately imply the following characterization of  $\gamma$ -compact sets.

**Proposition 2.4.** *Let  $X$  and  $Y$  be  $T_1$ -spaces and  $K \subseteq X \times Y$ . Then  $K$  is  $\gamma$ -compact if and only if when*

- (1)  $K$  is compact;
- (2)  $K \subseteq \text{cross}(C)$  for a finite set  $C \subseteq X \times Y$ .

### 3. CONNECTED SETS AND CROSS-MAPPINGS

**Proposition 3.1.** *Let  $X$  and  $Y$  be connected spaces,  $A$  a dense subset of  $X$ , let  $\emptyset \neq B \subseteq Y$  and  $C \subseteq X \times Y$  be such sets that  $\text{cross}(A \times B) \subseteq C$ . Then  $C$  is connected.*

*Proof.* Let  $U$  and  $V$  be open subsets of  $C$  such that  $C = U \sqcup V$ . Since  $X$  and  $Y$  are connected, for every  $p \in A \times B$  either  $\text{cross}(p) \subseteq U$ , or  $\text{cross}(p) \subseteq V$ . Since  $\text{cross}(p) \cap \text{cross}(q) \neq \emptyset$  for distinct points  $p, q \in X \times Y$ ,  $\text{cross}(A \times B) \subseteq U$  or  $\text{cross}(A \times B) \subseteq V$ . Taking into account that  $\text{cross}(A \times B)$  is dense in  $X \times Y$ , and consequently, in  $C$ , we obtain that  $C \subseteq U$  or  $C \subseteq V$ . Therefore,  $U = \emptyset$  or  $V = \emptyset$ . Hence,  $C$  is connected.  $\square$

**Corollary 3.2.** *Let  $X$  and  $Y$  be infinite connected  $T_1$ -spaces. Then the complement to any finite subset of  $X \times Y$  is connected.*

*Proof.* Let  $C \subseteq X \times Y$  be a finite set. We choose finite sets  $A \subseteq X$  and  $B \subseteq Y$  such that  $C \subseteq A \times B$ . Remark that  $A_1 = X \setminus A$  and  $B_1 = Y \setminus B$  are dense in  $X$  and  $Y$ , respectively, and  $\text{cross}(A_1 \times B_1) \subseteq (X \times Y) \setminus C$ . It remains to apply Proposition 3.1.  $\square$

**Definition 3.3.** A topological space  $X$  is said to be a  $C_1$ -space (or a space with the property  $C_1$ ), if the complement to any finite subset has finite many components.

Let us observe that the real line  $\mathbb{R}$  has the property  $C_1$ . Moreover, a finite product of  $C_1$ -spaces is a  $C_1$ -space.

Let  $X$  and  $Y$  be topological spaces and  $P \subseteq X \times Y$ . A mapping  $f : P \rightarrow X \times Y$  is called a *cross-mapping*, if  $f(p) \subseteq \text{cross}(p)$  for every  $p \in P$ .

**Lemma 3.4.** *Let  $X$  and  $Y$  be Hausdorff spaces,  $U \subseteq X$ ,  $V \subseteq Y$ , let  $f : U \times V \rightarrow X \times Y$  be a continuous cross-mapping, let  $A \subseteq X$  and  $B \subseteq Y$  be finite sets and the following conditions hold:*

- (1)  $U, V$  be connected  $C_1$ -spaces;
- (2)  $f(U \times V) \subseteq \text{cross}(A \times B)$ .

Then either  $f(U \times V) \subseteq \{a\} \times Y$  for some  $a \in A$ , or  $f(U \times V) \subseteq X \times \{b\}$  for some  $b \in B$ .

*Proof.* If both  $U$  and  $V$  are finite, then (1) imply that  $U$  and  $V$  are one-point sets and the lemma follows from (2). If  $U$  is finite (one-point) and  $V$  is infinite, then  $F = \{z \in U \times V : f(z) \in \text{cross}(A \times B) \setminus (A \times Y)\}$  is finite clopen subset of  $U \times V$ . The connectedness of  $U \times V$  implies  $F = \emptyset$ . Hence,  $f(U \times V) \subseteq A \times Y$ . Since  $f$  is continuous,  $f(U \times V) \subseteq \{a\} \times Y$  for some  $a \in A$ .

Now let  $U$  and  $V$  be infinite. Then it follows from (1) that  $U$  and  $V$  have no isolated points. Since  $A_1 = A \cap U$  and  $B_1 = B \cap V$  are closed and nowhere dense in  $U$  and  $V$ , respectively, the set

$$C = (U \times V) \cap \text{cross}(A \times B) = (U \times V) \cap \text{cross}(A_1 \times B_1)$$

is closed and nowhere dense in  $Z = U \times V$ .

Let  $\alpha : U \times V \rightarrow X$  and  $\beta : U \times V \rightarrow Y$  be continuous functions such that  $f(x, y) = (\alpha(x, y), \beta(x, y))$  for all  $(x, y) \in Z$ . Put

$$Z_\alpha = \{(x, y) \in Z : \alpha(x, y) = x\}, \quad Z_\beta = \{(x, y) \in Z : \beta(x, y) = y\}.$$

Notice that

$$P_\alpha = \{z \in Z_\alpha : \alpha(z) \in A\} = Z_\alpha \cap (A \times Y) = Z_\alpha \cap (A_1 \times Y)$$

is nowhere dense in  $Z$ . Therefore,  $Q_\alpha = \{z \in Z_\alpha : \alpha(z) \notin A\}$  is dense in  $\text{int}_Z(Z_\alpha)$ , where by  $\text{int}_Z(D)$  we denote the interior of  $D \subseteq Z$  in  $Z$ , and by  $\overline{D}$  we denote the closure of  $D$  in  $Z$ . Condition (2) implies that  $Q_\alpha$  is contained in the closed set  $\{z \in Z : \beta(z) \in B\}$ . Hence,

$$\overline{\text{int}_Z(Z_\alpha)} \subseteq \overline{Q_\alpha} \subseteq \{z \in Z : \beta(z) \in B\},$$

i.e.  $f(\overline{\text{int}_Z(Z_\alpha)}) \subseteq X \times B$ . Similarly,  $f(\overline{\text{int}_Z(Z_\beta)}) \subseteq A \times Y$ .

Since  $f$  is a cross-mapping,  $Z = Z_\alpha \cup Z_\beta$ . Remark that  $Z_\alpha$  and  $Z_\beta$  are closed in  $Z$ . Let

$$G = Z \setminus C.$$

Taking into account that  $C$  is closed and nowhere dense in  $Z$ , we have that  $G$  is open and dense in  $Z$ . According to (1) the sets  $U \setminus A$  and  $V \setminus B$  has finite many components, therefore, the set  $G = (U \setminus A) \times (V \setminus B)$  has finite many components  $G_1, \dots, G_k$ . Then  $G = \bigsqcup_{i=1}^k G_i$ , where all  $G_i$  are closed in  $G$ . Hence, all the sets  $G_i$  are clopen in  $G$ , in particular, open in  $Z$ . Notice that  $Z_\alpha \cap Z_\beta = \{z \in Z : f(z) = z\} \subseteq f(Z) \subseteq \text{cross}(A \times B)$ . Thus,  $G \cap Z_\alpha \cap Z_\beta = \emptyset$ . Then  $G_i \subseteq (Z_\alpha \cap G_i) \sqcup (Z_\beta \cap G_i)$ , consequently  $G_i \subseteq Z_\alpha$  or  $G_i \subseteq Z_\beta$  for every  $1 \leq i \leq k$ . Let

$$I_\alpha = \{1 \leq i \leq k : G_i \subseteq Z_\alpha\}, \quad I_\beta = \{1 \leq i \leq k : G_i \subseteq Z_\beta\},$$

$$U_\alpha = \bigcup_{i \in I_\alpha} G_i, \quad U_\beta = \bigcup_{i \in I_\beta} G_i.$$

Remark that

$$f(\overline{U_\alpha}) \subseteq f(\overline{\text{int}_Z(Z_\alpha)}) \subseteq X \times B, \quad f(\overline{U_\beta}) \subseteq f(\overline{\text{int}_Z(Z_\beta)}) \subseteq A \times Y.$$

Hence, for any  $z = (x, y) \in \overline{U_\alpha}$  we have  $\alpha(x, y) = x$  and  $\beta(x, y) \in B$ . Similarly,  $\alpha(x, y) \in A$  and  $\beta(x, y) = y$  for every  $z = (x, y) \in \overline{U_\beta}$ . Therefore,  $z = f(z) \in A \times B$  for any  $z \in \overline{U_\alpha} \cap \overline{U_\beta}$ . Consequently,  $Z_0 = \overline{U_\alpha} \cap \overline{U_\beta}$  is finite.

Denote  $E = \overline{U_\alpha} \setminus Z_0$  and  $D = \overline{U_\beta} \setminus Z_0$ . Since by Proposition 3.1 the set  $Z \setminus Z_0$  is connected, nonempty and  $Z \setminus Z_0 = E \sqcup D$ , taking into account that  $\overline{E} \cap D = \emptyset$  and  $E \cap \overline{D} = \emptyset$ , we obtain  $E = \emptyset$  or  $D = \emptyset$ . Assume that  $E = \emptyset$ . Then  $U_\beta$  is dense in  $Z$  and

$$f(Z) \subseteq f(\overline{U_\beta}) \subseteq A \times Y.$$

Since  $U \times V$  is connected,  $f(U \times V)$  is connected too, therefore, there is such  $a \in A$  that  $f(U \times V) \subseteq \{a\} \times Y$ .  $\square$

#### 4. THE MAIN RESULT

**Proposition 4.1.** *Let  $X$  and  $Y$  be  $T_1$ -spaces,  $z_0 \in X \times Y$  and let  $(z_n)_{n=1}^\infty$  be  $\gamma$ -convergent to  $z_0$  sequence of points  $z_n = (x_n, y_n) \in X \times Y$ . Then there exists  $m \in \mathbb{N}$  such that  $z_n \in \text{cross}(z_0)$  for all  $n \geq m$ .*

*Proof.* Assume the contrary. Then by the induction on  $k \in \mathbb{N}$  it is easy to construct a strictly increasing sequence of numbers  $n_k \in \mathbb{N}$  such that  $x_{n_i} \neq x_{n_j}$  and  $y_{n_i} \neq y_{n_j}$  for distinct  $i, j \in \mathbb{N}$ , and  $z_{n_k} \notin \text{cross}(z_0)$  for all  $k \in \mathbb{N}$ . Now the sequence  $(p_k)_{k=1}^\infty$  of points  $p_k = z_{n_k}$  converges to  $z_0$ , and from the other side the set  $G = (X \times Y) \setminus \{p_k : k \in \mathbb{N}\}$  is a neighborhood of  $z_0$ , a contradiction.  $\square$

A system  $\mathcal{A}$  of subsets of a topological space  $X$  is called a  $\pi$ -pseudobase [8], if for every nonempty open set  $U \subseteq X$  there exists a set  $A \in \mathcal{A}$  such that  $\text{int}(A) \neq \emptyset$  and  $A \subseteq U$ .

**Theorem 4.2.** *Let  $X$  and  $Y$  be Hausdorff spaces without isolated points and let  $X$  and  $Y$  have  $\pi$ -pseudobases which consist of connected compact  $C_1$ -sets, and let  $f : X \times Y \rightarrow X \times Y$  be the identical mapping. Then  $f \notin B_1(X \times Y, (X \times Y, \gamma))$ .*

*Proof.* Assuming the contrary, we choose a sequence of continuous functions  $f_n : X \times Y \rightarrow (X \times Y, \gamma)$  such that  $f_n(x, y) \rightarrow (x, y)$  in  $(X \times Y, \gamma)$  for all  $(x, y) \in X \times Y$ .

Remark that every  $f_n : X \times Y \rightarrow X \times Y$  is continuous. Then for every  $n \in \mathbb{N}$  the set  $P_n = \{p \in X \times Y : f_n(p) \in \text{cross}(p)\}$  is closed. Hence, for every  $n \in \mathbb{N}$  the set

$$F_n = \bigcap_{m \geq n} P_m = \{p \in X \times Y : \forall m \geq n \ f_m(p) \in \text{cross}(p)\}.$$

is closed too. Moreover, by Proposition 4.1

$$X \times Y = \bigcup_{n=1}^{\infty} F_n.$$

The conditions of the theorem imply that  $Z = X \times Y$  has a  $\pi$ -pseudobase of compact sets. Then the product contains an open everywhere dense locally compact subspace, in particular, the product  $X \times Y$  is Baire. We choose a number  $n_0 \in \mathbb{N}$  and compact connected  $C_1$ -sets  $U \subseteq X$  and  $V \subseteq Y$  such that  $U \times V \subseteq F_{n_0}$ ,  $U_0 = \text{int}(U) \neq \emptyset$  and  $V_0 = \text{int}(V) \neq \emptyset$ .

Let  $W = U \times V$ . According to Proposition 2.4, there exist such sequences of finite sets  $A_n \subseteq X$  and  $B_n \subseteq Y$  that  $f_n(W) \subseteq (A_n \times Y) \cup (X \times B_n)$  for every  $n \in \mathbb{N}$ . Since  $X$  and  $Y$  have no isolated points, the sets  $U_0$  and  $V_0$  are infinite. Take such points  $p_1 = (x_1, y_1), p_2 = (x_2, y_2) \in U_0 \times V_0$  that  $p_1 \notin \text{cross}(p_2)$ . Since  $X$  and  $Y$  are Hausdorff, there exist neighborhoods  $U_1$  and  $U_2$  of  $x_1$  and  $x_2$  in  $U_0$  and neighborhoods  $V_1$  and  $V_2$  of  $y_1$  and  $y_2$  in  $V_0$ , respectively, such that  $U_1 \cap U_2 = V_1 \cap V_2 = \emptyset$ . Now we choose a number  $N \geq n_0$  such that  $f_N(p_1) \in U_1 \times V_1$  and  $f_N(p_2) \in U_2 \times V_2$ .

Since  $f_N|_W$  is a cross-mapping, Lemma 3.4 implies that  $f_N(W) \subseteq \{a\} \times Y$  for some  $a \in A$  or  $f_N(W) \subseteq X \times \{b\}$  for some  $b \in B$ . Assume that  $f_N(W) \subseteq \{a\} \times Y$  for some  $a \in X$ . Then  $(U_1 \times V_1) \cap (\{a\} \times Y) \neq \emptyset$  and  $(U_2 \times V_2) \cap (\{a\} \times Y) \neq \emptyset$ . Therefore,  $a \in U_1 \cap U_2$ , which is impossible.  $\square$

**Corollary 4.3.** *Let  $n, m \geq 1$  and let  $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n \times \mathbb{R}^m$  be the identical mapping. Then  $f \notin B_1(\mathbb{R}^n \times \mathbb{R}^m, (\mathbb{R}^n \times \mathbb{R}^m, \gamma))$ .*

**Corollary 4.4.** *The collection  $(\mathbb{R}^n, \mathbb{R}^m, (\mathbb{R}^n \times \mathbb{R}^m, \gamma))$  is not a Lebesgue triple for all  $n, m \geq 1$ .*

## 5. SEPARATELY CONTINUOUS MAPPINGS ON ZERO-DIMENSIONAL SPACES

Recall that a nonempty topological space  $X$  is *strongly zero-dimensional*, if it is completely regular and every finite functionally open cover of  $X$  has a finite disjoint open refinement [9, . 529].

**Theorem 5.1.** *Let  $X$  be a strongly zero-dimensional metrizable space, let  $Y$  and  $Z$  be topological spaces. Then  $(X, Y, Z)$  is a Lebesgue triple.*

*Proof.* Let  $d$  be a metric on  $X$ , which generates its topology. For every  $n \in \mathbb{N}$  we consider an open cover  $\mathcal{B}_n$  of  $X$  by balls of the diameter  $\leq \frac{1}{n}$ . It follows from [7] that every  $\mathcal{B}_n$  has locally finite clopen refinement  $\mathcal{U}_n = (U_{\alpha,n} : 0 \leq \alpha < \beta_n)$ . For all  $n \in \mathbb{N}$  let  $V_{0,n} = U_{0,n}$  and  $V_{\alpha,n} = U_{\alpha,n} \setminus \bigcup_{\xi < \alpha} U_{\xi,n}$  if  $\alpha > 0$ . Then  $\mathcal{V}_n = (V_{\alpha,n} : 0 \leq \alpha < \beta_n)$  is a locally finite disjoint cover of  $X$  by clopen sets  $V_{\alpha,n}$  which refines  $\mathcal{B}_n$ .

Let  $f : X \times Y \rightarrow Z$  be a separately continuous function. For all  $n \in \mathbb{N}$  and  $0 \leq \alpha < \beta_n$  we choose a point  $x_{\alpha,n} \in V_{\alpha,n}$ . Let us consider functions  $f_n : X \times Y \rightarrow Z$  defined as the following:

$$f_n(x, y) = f(x_{\alpha,n}, y),$$

if  $x \in V_{\alpha,n}$  and  $y \in Y$ . Clearly, for every  $n \in \mathbb{N}$  the function  $f_n$  is jointly continuous, provided  $f$  is continuous with respect to the second variable. We show that  $f_n(x, y) \rightarrow f(x, y)$  on  $X \times Y$ . Fix  $(x, y) \in X \times Y$  and choose a sequence  $(\alpha_n)_{n=1}^\infty$  such that  $x \in V_{\alpha_n,n}$ . Since  $\text{diam} V_{\alpha_n,n} \rightarrow 0$ ,  $x_{\alpha_n,n} \rightarrow x$ . Taking into account that  $f$  is continuous with respect to the first variable, we obtain that

$$f_n(x, y) = f(x_{\alpha_n,n}, y) \rightarrow f(x, y).$$

Hence,  $f \in B_1(X \times Y, Z)$ . □

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